MATH 230 Homework #1 SOLUTIONS

1. $A=\{\text{no box is empty}\}$ $n(A)=\binom{n-1}{n-1}=1$ $n(S)=\binom{n+n-1}{n-1}=\binom{2n-1}{n-1}$

$$P(A)=\frac{1}{\binom{2n-1}{n-1}}$$

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2. $n(S)=900$ a) $A'=\{\text{There is no 5 in selected number}\}$ $8\cdot 9\cdot 9$ (no "5", no "0")

$P(A')=\frac{648}{900}$ $P(A)=P(\text{at least one 5})=1-\frac{648}{900}=0.28$

b) $B=\{\text{exactly 2 5's}\}$ $n(B)=8+9+9=26$ $P(B)=\frac{26}{900}$

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3. $P(A)=0.80$, $P(B)=0.20$ $D=\{\text{defective part}\}$ $P(D|A)=0.03$, $P(D|B)=0.05$

$P(A|D)=?$ $P(B|D)=?$ $P(D)=P(A).P(D|A)+P(B)P(D|B)$

$=(0.8)(0.03)+(0.2)(0.05)=0.024+0.01=0.034$

$P(A|D)=\frac{0.024}{0.034}=0.706$ $P(B|D)=1-0.706=0.294$ A is more likely to have supplied a defective part.

Because $P(A|D)>P(B|D)$.

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4. X: # of available channels

x	0	1	2	3	4	5
$f(x)$	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$

$$E(X)=0\tfrac{1}{32}+1\tfrac{5}{32}+2\tfrac{10}{32}+3\cdot\tfrac{10}{32}+4\cdot\tfrac{5}{32}+5\cdot\tfrac{1}{32}=2.5$$

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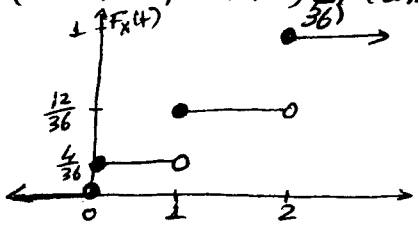
5. $P(\text{win})=P(\text{sum is 7 or 11})=\frac{8}{36}$, $P(\text{lose})=P(\text{sum is 2, 3 or 12})=\frac{4}{36}$ $P(\text{cont. to play})=\frac{24}{36}$

$$f(x)=\begin{cases}\frac{4}{36} & x=0\\ \frac{8}{36} & x=1\\ \frac{24}{36} & x=2\end{cases}\qquad F_X(t)=\begin{cases}0 & t<0\\ \frac{4}{36} & 0\le t<1\\ \frac{12}{36} & 1\le t<2\\ 1 & t\ge 2\end{cases}$$



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6. X: # of shots needed to make the 1-st basket $X\sim$ Geometric $(p=0.7)$

a) $P(X<5)=\sum_{x=1}^{4}(0.3)^{x-1}(0.7)$

b) $P(X=1)+P(X=3)+P(X=5)+\cdots=0.7+(0.3)^2(0.7)+(0.3)^4(0.7)+\cdots$

$=(0.7)[1+(0.3)^2+(0.3)^4+\cdots]=0.7\cdot\frac{1}{1-(0.3)^2}=\frac{70}{81}$

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7. X: # of digit reversals in 10 digits $X\sim Bin(n=10,\ p=0.01)$

a) $P(X=0)=\binom{10}{0}(0.01)^0(0.99)^{10}=0.904$

b) Y: # of correct words in 5. $Y\sim Bin(n=5,\ p=0.904)$

$P(Y=5)=\binom{5}{5}(0.904)^5(0.096)^0=0.604$

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8. $X \sim Bin(n,p)$

$$M_X(t) = E(e^{tx}) = \sum_{x=0}^{n} e^{tx}\binom{n}{x} p^x q^{n-x} = \sum_{x=0}^{n} \binom{n}{x}(pe^t)^x q^{n-x}$$

$$M_X(t) = (pe^t + q)^n$$

$$m'_X(t)\Big|_{t=0} = npe^t(pe^t+q)^{n-1}\Big|_{t=0} = np[p+(1-p)]^{n-1} = np = E(X)$$

$$m''_X(t)\Big|_{t=0} = [npe^t(pe^t+q)^{n-1} + (n-1)np^2e^{2t}(pe^t+q)^{n-2}]_{t=0}$$

$$= np + n^2p^2 - np^2 = E(X^2)$$

$$Var(X) = np + n^2p^2 - np^2 - n^2p^2 = np(1-p) = npq$$

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